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Covariance of Light-Front Models: Pair Current

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Abstract

We compute the + component, *i.e.*, $j^+ = j^0 + j^3$, of the electromagnetic current of a composite spin-one two-fermion system for vanishing momentum transfer component $q^+ = q^0 + q^3$. In particular, we extract the nonvanishing pair production amplitude on the light-front. It is a consequence of the longitudinal zero momentum mode, contributing to the light-front current in the Breit-frame. The covariance of the current is violated, if such pair terms are not included in its matrix elements. We illustrate our discussion with some numerical examples.

Key words: Electromagnetic currents, pair terms, relativistic quark model

1 Introduction

The light-front hypersurface given by $x^+ = t + z = 0$ (null-plane) is adequate for defining a light-front wave-function, as suggested by Dirac [1]. The

null-plane is invariant under seven kinematical transformations, three of them include boosts. It is possible to keep a limited covariance of physical observables under kinematical boosts, within a fixed number of degrees of freedom [2]. However, the rotations around the x- and y-axis do not belong to the stability group; the corresponding transformations are not kinematic and therefore particle-antiparticle pair creation is possible. Thus, the description of a physical system in terms of a composite null-plane wave-function with a minimal, fixed number of particles in general violates complete covariance.

The light-front electromagnetic (e.m.) current j of a composite system can in principle be constructed from covariant Feynman diagrams. In the Breit-frame with momentum transfer $q^+ = q^0 + q^3 = 0$, the one-loop expressions can be integrated over the k^- ($k^- = k^0 - k^3$) component of the internal loop momentum [3–6]. This procedure can be compared to the exact relation of the "old-fashioned" perturbation theory with the covariant perturbative expansion of the field theoretical S-matrix. In a series of papers by Chang, Root, and Yan [7], it was demonstrated that the perturbative expansion of light-front field theory is obtained from the covariant Feynman amplitudes by first integrating over k^- in the momentum loops. However, in some cases the naive Cauchy integration of the k^- momentum gave completely wrong results [7]. The connection between light-front and covariant field theory was further developed in the recent works by Ligterink and Bakker [8] and Schoonderwoerd and Bakker [9], with special attention to the separation of the zero-mode in the perturbative amplitudes. The failure of the naive Cauchy integration amounts to neglecting the longitudinal zero-momentum modes [6,8,10–12].

In this paper, we discuss the current of a vector particle, treated as a composite fermion system. We explicitly calculate the Feynman one-loop triangle diagram for two different couplings of the fermions to the vector particle. In [5] it has been shown that the current component j^+ of this spin-one system violates the requirements of covariance, parity and gauge invariance, the so called "angular condition" [13–16], when the naive Cauchy integration is performed in the triangle diagram for $q^+ = 0$. A complete elimination of the pairs created out or annihilating into the vacuum was believed to be possible, leading to a description in terms of a two-particle light-front wave-function [3–6]. Although such a current is covariant under kinematical boost transformations, it does not have the correct transformation properties under general rotations and parity transformations. It has been pointed out in an example with γ^μ-coupling, that this problem appears once a nonvanishing pair term around $q^+ = 0$ in the k^- integration is neglected [10].

Therefore we focus in this work on the difficulty in the k^- integration and its cure by a careful treatment of singularities. Our treatment is different and the result is more general than the one in Refs. [8]. In the case of the j^+ component, the longitudinal zero-mode contribution to the amplitude is equal to the

one for virtual pair creation by the photon with $q^+ = 0$ [10]. These light-front pair production contributions to the j^+ current of a vector composite particle at $q^+ = 0$ are derived using the method of "dislocation of pole integration", developed in Refs. [6,10,11]. The contributions originate from the instantaneous part of the light-front fermion propagator and from the derivative coupling of the composite particle to the fermions. We use the covariant model proposed in Ref.[5] for the rho-meson e.m. current, in which Pauli-Villars regulators are introduced in the triangle diagram. For comparision we also perform the covariant calculation which corresponds to directly integrating the momentum loop in the standard variables of the four-dimensional phase-space, *i.e.*, first integrating over k^0 analytically and then numerically in the remaining three-dimensional momentum space.

This paper is organized as follows. In the next section the electromagnetic current of a spin-one particle is discussed. In section III the method of dislocating poles is outlined. The model for the electromagnetic current is presented in section IV and the importance of including the pair terms in the light-front calculation is emphasized. In the last section we show our numerical results, discuss their implications and conclude with a brief summary.

2 Electromagnetic current

The general expression of the e.m. current of a spin-one particle has the form [17] :

$$j^{\mu}_{\alpha\beta} = \left[F_1(q^2)g_{\alpha\beta} - F_2(q^2)\frac{q_\alpha q_\beta}{2m_v^2}\right](P'+P)^\mu - F_3(q^2)\left[q_\alpha g^\mu_\beta - q_\beta g^\mu_\alpha\right] \ , \quad (1)$$

where m_v is the mass of the composite vector particle, q^μ the momentum transfer and $P^\mu(P'^\mu)$ is the initial (final) on-mass shell momentum.

The impulse approximation of the electromagnetic current J^μ is given by [5]:

$$j^{\mu}_{\alpha\beta} = \imath \int \frac{d^4k}{(2\pi)^4}\Lambda(k,P')\Lambda(k,P) \times \frac{Tr[\Gamma'_\alpha(\not k - \not P' + m)\gamma^\mu(\not k - \not P + m)\Gamma_\beta(\not k + m)]}{((k-P)^2 - m^2 + \imath\epsilon)(k^2 - m^2 + \imath\epsilon)((k-P')^2 - m^2 + \imath\epsilon)} \ , \quad (2)$$

where Γ'_α and Γ_β define the coupling of the fermions to the vector particle. The functions Λ are used to regularize the model, *i.e.*, render the momentum integrals finite. These Pauli-Villars regulators, was well as the couplings, will be explicitly defined in section IV.

The matrix elements

$$j^+_{ji} := \epsilon_j^{'\alpha} j^+_{\alpha\beta} \epsilon_i^\beta \ , \tag{3}$$

in the instant-form spin basis are obtained from Eq.(1) in the Breit-frame, where $q^\mu = (0, q_x, 0, 0)$. The initial instant-form cartesian polarization four-vectors of the spin-one particle are

$$\epsilon^\mu_x = (-\sqrt{\eta}, \sqrt{1+\eta}, 0, 0) \ , \ \epsilon^\mu_y = (0, 0, 1, 0) \ , \ \epsilon^\mu_z = (0, 0, 0, 1) \ , \tag{4}$$

with $\eta = -q^2/4m_v^2$. For the final polarization states we have

$$\epsilon^{'\mu}_x = (\sqrt{\eta}, \sqrt{1+\eta}, 0, 0) \ , \ \epsilon^{'\mu}_y = \epsilon^\mu_y \ , \ \epsilon^{'\mu}_z = \epsilon^\mu_z \ . \tag{5}$$

The vector particle four-momenta are $P^\mu = (P^0, -q_x/2, 0, 0)$ for the initial state and $P'^\mu = (P^0, q_x/2, 0, 0)$ for the final state; $P^0 = m_v\sqrt{1+\eta}$. In this frame, the nonzero matrix elements of j^+ are j^+_{xx}, j^+_{zx} $(= -j^+_{xz})$, j^+_{yy} and j^+_{zz}. The angular condition corresponds to the equality $j^+_{yy} = j^+_{zz}$ [14] and should be satisfied for the current matrix elements, Eq.(3), for the particular polarization vectors given above.

In the next section, we will present the analytical method that allows to integrate the particular terms of the impulse approximation of the current, Eq.(2), which contain the amplitude for pair production by the incoming photon.

3 The "dislocation of pole" integration

Here we explicitly consider the one-loop integration over the k^- momentum variable for arbitrary number of propagators and Pauli-Villars regulators [10,11]. We generalize the method that was derived to compute the pair production amplitude for the j^- component of the current of a composite boson in the limit of $q^+ \to 0$ [10]. The technique has also been succesfully applied in the context of a perturbative bosonic model in light-front field theory [11]. In a recent work, we have used this method in the calculation of both components, j^+ and j^-, of the electromagnetic current of the pseudoscalar pion. The relevance of pair terms for the j^- component of this current was demonstrated [6].

The integral in the variables k^+ and k^- is given by the building blocks

$$I^{(N)}_{mn}(\{f_i\}) = \int \frac{dk^+ dk^-}{2} \frac{(k^-)^m (P^+ - k^+)^n}{\prod_{j=1}^{N} (P^+ - k^+)(P^- - k^- - \frac{f_j - \imath\epsilon}{P^+ - k^+})} \ , \tag{6}$$

with external four-momentum components P^+ and P^- and $f_j > 0$. The parameters $m,\ n$ are limited to be $m \leq n$; the other integer N should be large enough to render the integral finite. Note that Eq.(6) would be zero by naive Cauchy integration over k^-. As we will prove, $I^{(N)}_{mn}(\{f_i\})$ does not depend on the specific values of $P^+ \ > \ 0$ and P^-.

In the denominator one of the N poles is dislocated via $P^+ \to P^+ + \delta \ \ (\delta > 0)$, *i.e.*,

$$\frac{1}{(P^+ - k^+)(P^- - k^-) - f_1 + i\epsilon} \to \frac{1}{(P^+ + \delta - k^+)} \frac{1}{P^- - k^- - \frac{f_1 - i\epsilon}{P^+ + \delta - k^+}} \ , \tag{7}$$

violating the symmetry with respect to the permutation of the factors j. However, it is shown below that the final results are symmetrical under permutation of f_j.

Nonanalytical terms in δ are not present after the k^- integration. Thus, the limit $\delta \to 0$ can be performed. Either the integration vanishes with positive integer power of δ or the infinitesimal parameter δ can be eventually absorbed by rescaling the variable k^+ $(x = (k^+ - P^+)/\delta)$, as done in [6,10,11]. The limit $\epsilon \to 0$ can also be taken and the results are:

$$I^{(N)}_{mn}(\{f_i\}) = 0 \ , \quad m \ < \ n \ , \tag{8}$$

and

$$I^{(N)}_{mm}(\{f_i\}) = \imath\pi \sum_{j=1}^{N} \frac{(-f_j)^m \ln(f_j)}{\prod_{k=1, k\neq j}^{N} (f_j - f_k)} \ , \quad m = n \ . \tag{9}$$

The first result, Eq.(8), immediately follows from Cauchy's theorem. The results contained in Eq.(9) are proved in the Appendix. The case of $I^{(N)}_{00}(\{f_i\})$ has also been obtained in Ref.[8] using a different technique. The form of Eq.(9) is again symmetrical under permutation of f_j, a posteriori justifying our choice in Eq.(7).

Using a model for the composite vector particle [5], we will show that a naive, incorrect evaluation of the integrals $I^{(N)}_{mn}(\{f_i\})$ causes the seeming violation of covariance in the form-factor calculation using j^+ in a frame with vanishing momentum transfer component, $q^+ = 0$. It is necessary to include pair production, implicitly given by Eq.(9), in order to get agreement with the covariant calculation. The interpretation of results obtained with the "dislocation of

pole" method, especially the identification of zero mode contributions as pair terms, was given in Refs.[6,10,11].

4 The model and discussion of pair terms

In order to discuss the "good" component j^+ of the current [18] of the vector particle, we specify the covariant model [5]. The composite spin-one particle is described as a two-fermion system and two forms of the coupling in Eq.(2) have been used:

$$i)\ \Gamma_\mu = \Gamma'_\mu = \gamma_\mu \ , \quad ii)\ \Gamma_\mu = (2k_\mu - P_\mu)\ ; \Gamma'_\mu = (2k_\mu - P'_\mu) \ , \tag{10}$$

where the primed quantities refer to the final state. For the regularization function $\Lambda(k, P)$, choosen to make the integral of Eq.(2) finite, we take $\Lambda(k, P) = (2\pi)^2 C \left[(k-P)^2 - m_R^2 + \imath\epsilon\right]^{-2}$. This choice serves our purpose of presenting the main points on how the pair current preserves the rotational invariance of j^+ on the light front. It also allows to identify a null-plane wave-function, when one looks at contribution of the residue due to the pole of the spectator particle for $k^- = \frac{k_\perp^2 + m^2 - \imath\epsilon}{k^+}$ with $0 < k^+ < P^+$ in the k^- integration of Eq.(2) [5]. The factor C is fixed by the charge normalization. The model fulfills current conservation [5].

The instantaneous part of the fermion propagator is the second term of the Dirac propagator decomposed using the light-front momenta

$$\frac{\not{k} + m}{k^2 - m^2 + \imath\epsilon} = \frac{\not{k}_{(on)} + m}{k^+(k^- - k^-_{(on)} + \frac{\imath\epsilon}{k^+})} + \frac{\gamma^+}{2k^+} \ , \tag{11}$$

where $k^-_{(on)} = \frac{k_\perp^2 + m^2}{k^+}$. The presence of a nonvanishing zero-mode contribution to j^+ is, in case of γ_μ-coupling Eq.(10 i) only due to the instantaneous part of the Dirac propagator. For derivative coupling Eq.(10 ii) the zero-mode contribution is caused by the vertices as well.

Eq.(2) yields the matrix elements of the current depending on the polarization states. We call the matrix elements of j^+, which do not couple to the zero-mode and correspond to the numerator of Eq.(2) being dependent only on k^+, $k_\perp$ and $(k^-)^{m+1}(P^+ - k^+)^n$ with $m < n$ in Eq.(8), "good" matrix elements. In such cases only the pole at which the spectator particle is on-mass-shell, contributes to the Cauchy integration over k^- in the limit of $\delta \to 0_+$. The pole in Eq.(2) is placed on the lower half of the complex k^- plane at $k^- = (k_\perp^2 + m^2 - \imath\epsilon)/k^+$ with $0 < k^+ < P^+$.

i) γ^μ-coupling

The instantaneous component of the fermion propagator, cf. Eq.(11), yields a nontrivial zero-mode coupling to j^+. This contribution can be separated in Eq.(2) and can be written in the following form

$$
\begin{aligned}
j^{+pair}_{xx} &= -\eta B_\gamma(q^2) \ , \\
j^{+pair}_{zx} &= -\sqrt{\eta} B_\gamma(q^2) \ , \\
j^{+pair}_{zz} &= B_\gamma(q^2) \ ,
\end{aligned} \tag{12}
$$

where

$$
B_\gamma(q^2) = 4\imath C^2 \int \frac{d^2k_\perp}{P^+} I^{(6)}_{00}(\{f_i\}) \left(k^2_\perp + m^2 + \frac{q^2}{4} \right) \ , \tag{13}
$$

with the functions f_i given by

$$
\begin{aligned}
& f_1 = (P-k)^2_\perp + m^2 \ ; \ f_2 = (P'-k)^2_\perp + m^2 \ ; \ f_3 = (P-k)^2_\perp + m^2_R \ ; \\
& f_4 = (P'-k)^2_\perp + m^2_R \ ; \ f_5 = f_3 \ ; \ f_6 = f_4 \ .
\end{aligned} \tag{14}
$$

ii) Derivative coupling

In this example, the virtual pair production amplitude by the $q^+ \to 0$ photon has two sources. The first one is the instantaneous part of the fermion propagator and the second one is the light-front time derivative in the vertex. The numerator in the expression for j^+, cf. Eq.(2) and Eq.(3), is written as:

$$
\begin{aligned}
& Tr \left[\Gamma'_\mu \epsilon'^\mu_j (\not{k} - \not{P}' + m) \gamma^+ (\not{k} - \not{P} + m) \Gamma_\nu \epsilon^\nu_i (\not{k} + m) \right] = \\
& (2k - P')_\mu \epsilon'^\mu_j (2k - P)_\nu \epsilon^\nu_i Tr \left[(\not{k} - \not{P}' + m) \gamma^+ (\not{k} - \not{P} + m)(\not{k} + m) \right] .
\end{aligned} \tag{15}
$$

The pair production amplitude can now be found from Eq.(15). We first collect the terms that depend on powers of k^-, use $I^{(N)}_{mn} = 0$ for $m < n$, and then insert the identity

$$
\frac{(k^-)^2 k^+}{k^2 - m^2 + i\epsilon} = (k^- - P^-) + P^- + \frac{k^2_\perp + m^2}{k^+} + \frac{k^-_{(on)}(k^2_\perp + m^2)}{k^2 - m^2 + i\epsilon} \ . \tag{16}
$$

This identity is used, whenever the factor $(k^-)^2 k^+$ appears without being multiplied by powers of $(P^+ - k^+)$. The first term in the r.h.s of Eq.(16) is odd under the transformation $(P^- - k^-) \to -(P^- - k^-)$ and thus its

contribution to the pair current is zero. The last term of the r.h.s. of Eq.(16) also does not contribute to the pair current. The final result is

$$\begin{aligned} j^{+pair}_{xx} &= \eta B_D(q^2) \ , \\ j^{+pair}_{zx} &= \sqrt{\eta} B_D(q^2) \ , \\ j^{+pair}_{zz} &= -B_D(q^2) \ , \end{aligned} \tag{17}$$

where $B_D(q^2)$ is given in terms of the functions f_i, cf. Eq.(14),

$$\begin{aligned} B_D(q^2) = 4\imath C^2 \int \frac{d^2 k_\perp}{P^+} \{ & I^{(6)}_{22}(\{f_i\}) + 2 I^{(6)}_{11}(\{f_i\})(k^2_\perp + m^2) + \\ & I^{(6)}_{00}(\{f_i\}) \left(k^2_\perp + m^2 + \frac{q^2}{4} \right) \left(k^2_\perp + m^2 - m^2_v(1+\eta) \right) \} \ . \end{aligned} \tag{18}$$

The following discussion applies to both cases, *i.e.*, γ^μ-coupling and derivative coupling. The matrix elements of the good component j^+ in Eq.(2) are given by the sum of two terms: $j^+ = j^{+wf} + j^{+pair}$. The first one, j^{+wf}, is the contribution of the light-front wave-function, with k^+ integrated in the range $0 < k^+ < P^+$ in Eq.(2) [5]. The second one stems from the pair term, j^{+pair}, obtained with Eqs.(12, 17). Each matrix element of the current acquires a contribution from the pair term with the exception of j^+_{yy}, *i.e.*, $j^{+pair}_{yy} = 0$. In other words, the longitudinal zero mode contributes to j^+_{xx}, j^+_{zx} and j^+_{zz}.

The angular condition in the Cartesian spin basis is

$$\Delta(q^2) = j^+_{yy} - j^+_{zz} = j^{+wf}_{yy} - j^{+wf}_{zz} - j^{+pair}_{zz} = 0 \ . \tag{19}$$

Since the matrix elements of the current j^+_{ji} are obtained from a covariant model, $\Delta(q^2)$ should indeed be identical zero. Below, cf. Figs.4-5, we will explicitly confirm this in our calculations. Considering only the residue at the pole of the spectator particle in the Cauchy integration over the longitudinal momentum implies the violation of the angular condition. This has also been shown in a recent numerical study of this model [5].

It is due to the omission of the pair contribution to j^+_{zz}, *i.e.*, $j^{+pair}_{zz} = j^{+wf}_{yy} - j^{+wf}_{zz}$. The rotational symmetry of the "good" component of the current j^+ for a spin-one particle is restored, if the pair creation process is carefully taken into account in the computation of the Breit-frame matrix elements. The "good" component of the current receives a contribution from the pair term for $q^+ = 0$, and thus it is necessary to go beyond the naive light-front integration in k^-. One should include the zero-mode contribution to the current in order to calculate consistently e.m. form-factors in this reference frame.

5 Numerical results and summary

To illustrate the previous discussion, we compare the numerical results for the matrix elements of j^+ as function of the momentum transfer q^2 obtained in the covariant formalism [5] to the ones obtained in the light-front framework – with and without pair terms. We present the results for γ^μ-coupling and derivative coupling. The parameters of the model are taken from Ref. [5] and are $m_v = 0.77$ GeV, $m = 0.43$ GeV and $m_R = 1.8$ GeV.

Fig. 1 contains the results for j^+_{xx}. For both couplings, sizeable deviations between covariant – and light-front calculation, *not* taking into account pair terms, show up. Especially the derivative coupling yields a pronounced difference. This fact is due to the higher powers of k^- in the numerator of Eq.(2) as compared to the γ^μ-coupling that has k^- up to the first order. For $q^2 \to 0$, the zero mode contribution vanishes as q^2. We performed the calculation of the pair current, j^{+pair}_{xx}, according to Eq.(12) (γ^μ-coupling) and Eq.(17) (derivative coupling). In both cases, including the pair amplitude in the light-front current reproduces the covariant results within our numerical precision.

The same comparisons are made for the current component j^+_{zx} in Fig.2 and for j^+_{zz} in Fig.3. Here we also observe that the inclusion of pair production terms in the light-front computation yields results in agreement with the covariant ones. For small momentum transfer, the pair term in j^+_{zx} is proportional to q_x. However, even the slope of the curve shows zero-mode effects in this limit. Increasing the momentum transfer, the zero-mode contribution tends to be less important. In Figs.4-5, we show the angular condition, given by Eq.(19), for γ^μ-coupling and derivative coupling, respectively. If the pair current is included in the light-front calculation, this condition is fulfilled. This means that j^+_{yy} and j^+_{zz} indeed coincide. Omission of the light-front pair terms, however, causes a violation of the angular condition. Note that even at $q_x = 0$, the zero-mode contribution is nonvanishing in j^+_{zz}. The numerical results not only determine the magnitude of the effects arising from the pair current but also give further support to our analytical findings that the pair contribution is required to regain the covariant result.

In summary, using a fermion one-loop model we have derived the contribution of the light-front pair amplitude to the e.m. current of a composite vector particle in a reference frame where $q^+ = 0$. We have shown that the "good" component of the current j^+ gets contributions of the longitudinal zero-momentum mode. Our discussion is based on a careful integration, exploiting the recently developed "dislocation of pole" technique, in the momentum-loop of the triangle diagram for the virtual photon absorption process. For two examples of the vector particle–fermion coupling, we have analytically and numerically calculated the pair current.

It is shown that it is not possible to leave out that contribution without violating Lorentz covariance and, consequently, the angular condition in the Breit frame.

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APPENDIX

In this Appendix, we prove Eq.(9); first we consider the case $m = n = 0$ and use induction in N. Suppose that

$$\begin{aligned} I^{(N)}_{00}(\{f_i\}) &= \frac{1}{2}\int \frac{dk^+dk^-}{\prod_{j=1}^N (P^+ - k^+)(P^- - k^- - \frac{f_j - \imath\epsilon}{P^+ - k^+})} \\ &= \imath\pi \sum_{j=1}^N \frac{\ln(f_j)}{\prod_{k=1,k\neq j}^N (f_j - f_k)} \ , \end{aligned} \tag{20}$$

is valid for some $N \geq 2$; below we will show its validity for $N+1$.

We begin by explicitly verifying the identity for $N = 2$

$$\begin{aligned} & I^{(2)}_{00}(\{f_i\}) = \\ &= \frac{1}{2}\int \frac{dk^+dk^-}{(P^+ + \delta - k^+)(P^+ - k^+)(P^- - k^- - \frac{f_1 - \imath\epsilon}{P^+ + \delta - k^+})(P^- - k^- - \frac{f_2 - \imath\epsilon}{P^+ - k^+})} \\ &= \imath\pi \frac{\ln(f_1) - \ln(f_2)}{f_1 - f_2} \ . \end{aligned} \tag{21}$$

Next we use that:

$$\begin{aligned} & \frac{1}{(P^+ - k^+)(P^- - k^- - \frac{f_N - \imath\epsilon}{P^+ - k^+})} \times \frac{1}{(P^+ - k^+)(P^- - k^- - \frac{f_{N+1} - \imath\epsilon}{P^+ - k^+})} = \frac{1}{f_N - f_{N+1}} \\ & \times \left(\frac{1}{(P^+ - k^+)(P^- - k^- - \frac{f_N - \imath\epsilon}{P^+ - k^+})} - \frac{1}{(P^+ - k^+)(P^- - k^- - \frac{f_{N+1} - \imath\epsilon}{P^+ - k^+})} \right) . \end{aligned} \tag{22}$$

With the aid of Eq.(21), we write:

$$
\begin{aligned}
& I_{00}^{(N+1)}(\{f_i\}) = \\
&= \frac{i\pi}{f_N - f_{N+1}} \left[\sum_{j=1}^{N} \frac{\ln(f_j)}{\prod_{k=1,k\neq j}^{N}(f_j - f_k)} - \sum_{j=1,j\neq N}^{N+1} \frac{\ln(f_j)}{\prod_{k=1,k\neq j,k\neq N}^{N}(f_j - f_k)} \right] \\
&= \frac{i\pi}{f_N - f_{N+1}} \left[\sum_{j=1,j\neq N+1}^{N+1} \frac{\ln(f_j)}{\prod_{k=1,k\neq j,k\neq N+1}^{N+1}(f_j - f_k)} - \sum_{j=1,j\neq N}^{N+1} \frac{\ln(f_j)}{\prod_{k=1,k\neq j,k\neq N}^{N}(f_j - f_k)} \right] \\
&= \frac{i\pi}{f_N - f_{N+1}} \left[\sum_{j=1,j\neq N+1}^{N+1} \frac{\ln(f_j)(f_j - f_{N+1})}{\prod_{k=1,k\neq j}^{N+1}(f_j - f_k)} - \sum_{j=1,j\neq N}^{N+1} \frac{\ln(f_j)(f_j - f_N)}{\prod_{k=1,k\neq j}^{N}(f_j - f_k)} \right] \\
&= \frac{i\pi}{f_N - f_{N+1}} \sum_{j=1,}^{N+1} \frac{\ln(f_j)(f_j - f_{N+1} - f_j + f_N)}{\prod_{k=1,k\neq j}^{N+1}(f_j - f_k)} \\
&= i\pi \sum_{j=1}^{N+1} \frac{\ln(f_j)}{\prod_{k=1,k\neq j}^{N+1}(f_j - f_k)} \ .
\end{aligned} \tag{23}
$$

Secondly, we prove Eq.(9) by induction in m. Suppose that

$$
\begin{aligned}
I_{mm}^{(N)}(\{f_i\}) &= \int \frac{dk^+dk^-}{2} \frac{(k^-)^m(P^+ - k^+)^m}{\prod_{j=1}^{N}(P^+ - k^+)(P^- - k^- - \frac{f_j - \imath\epsilon}{P^+ - k^+})} \\
&= \imath\pi \sum_{j=1}^{N} \frac{(-f_j)^m \ln(f_j)}{\prod_{k=1,k\neq j}^{N}(f_j - f_k)} \ ,
\end{aligned} \tag{24}
$$

is valid for m, then consider

$$
\begin{aligned}
& I_{m+1\ m+1}^{(N)}(\{f_i\}) = \int \frac{dk^+dk^-}{2} \times \\
& \frac{(k^- - P^-)(k^-)^m(P^+ - k^+)^{m+1} + P^-(k^-)^m(P^+ - k^+)^{m+1}}{\prod_{j=1}^{N}(P^+ - k^+)(P^- - k^- - \frac{f_j - \imath\epsilon}{P^+ - k^+})} \ .
\end{aligned} \tag{25}
$$

The second term in the integrand of Eq.(25) vanishes due to $I_{mn}^{(N)}(f_i) = 0$ for $m < n$. Then we obtain

$$
\begin{aligned}
& I_{m+1\ m+1}^{(N)}(\{f_i\}) = \\
&= -\int \frac{dk^+dk^-}{2} \frac{\left((P^- - k^-)(P^+ - k^+) - f_1 + f_1\right)(k^-)^m(P^+ - k^+)^m}{\prod_{j=1}^{N}(P^+ - k^+)(P^- - k^- - \frac{f_j - \imath\epsilon}{P^+ - k^+})} \\
&= -\int \frac{dk^+dk^-}{2} \frac{(k^-)^m(P^+ - k^+)^m}{\prod_{j=1,j\neq 1}^{N}(P^+ - k^+)(P^- - k^- - \frac{f_j - \imath\epsilon}{P^+ - k^+})}
\end{aligned}
$$

$$
\begin{aligned}
&-f_1 \int \frac{dk^+ dk^-}{2} \frac{(k^-)^m (P^+ - k^+)^m}{\prod_{j=1}^N (P^+ - k^+)(P^- - k^- - \frac{f_j - \imath\epsilon}{P^+ - k^+})} \\
&= -\imath\pi \left[\sum_{j=1, j\neq 1}^{N} \frac{(-f_j)^m \ln(f_j)}{\prod_{k=1,k\neq j,k\neq 1}^N (f_j - f_k)} + \sum_{j=1}^{N} \frac{f_1 (-f_j)^m \ln(f_j)}{\prod_{k=1,k\neq j}^N (f_j - f_k)} \right] \\
&= -\imath\pi \left[\sum_{j=1}^{N} \frac{(f_j - f_1)(-f_j)^m \ln(f_j)}{\prod_{k=1,k\neq j}^N (f_j - f_k)} + \sum_{j=1}^{N} \frac{f_1 (-f_j)^m \ln(f_j)}{\prod_{k=1,k\neq j}^N (f_j - f_k)} \right] \\
&= \imath\pi \sum_{j=1}^{N} \frac{(-f_j)^{m+1} \ln(f_j)}{\prod_{k=1,k\neq j}^N (f_j - f_k)}, \qquad (26)
\end{aligned}
$$

which completes the proof.

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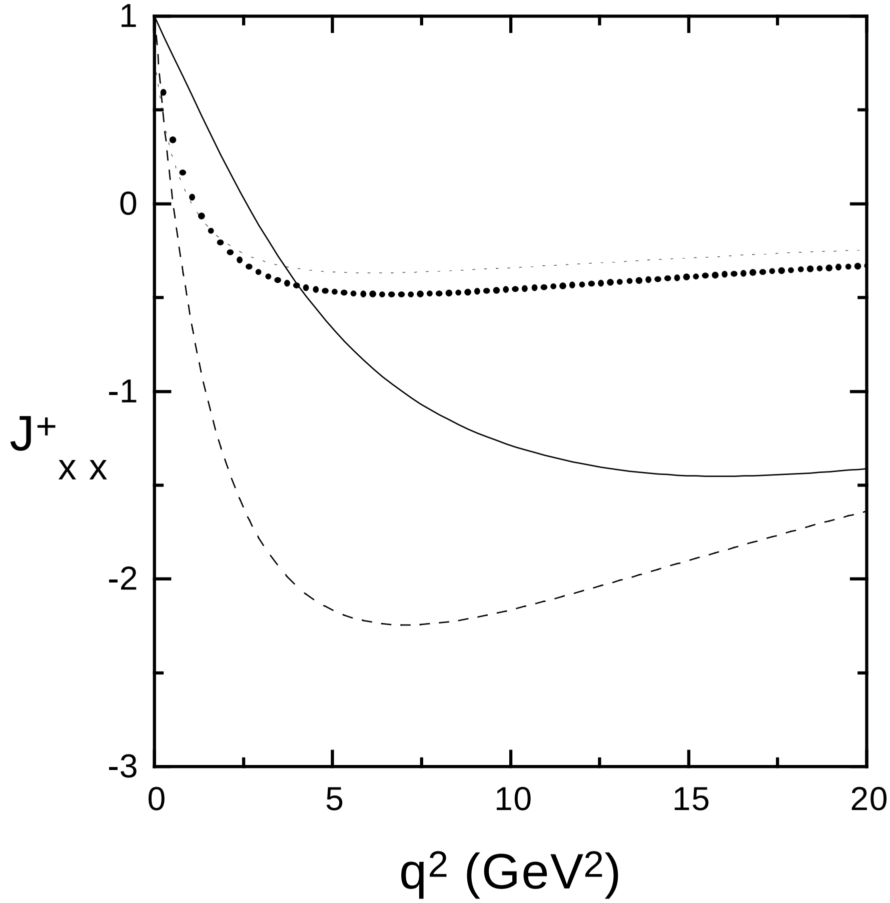

Fig. 1. Current component j^+_{xx} as a function of $|q^2|$. Results for γ^μ-coupling: covariant and light-front including pair current (full circles), light-front without pair current (dotted curve). Results for derivative coupling: covariant and light-front including pair current (solid curve), light-front without pair current (dashed curve). The results for the covariant calculation and for the light-front calculation including pair current contribution coincide.

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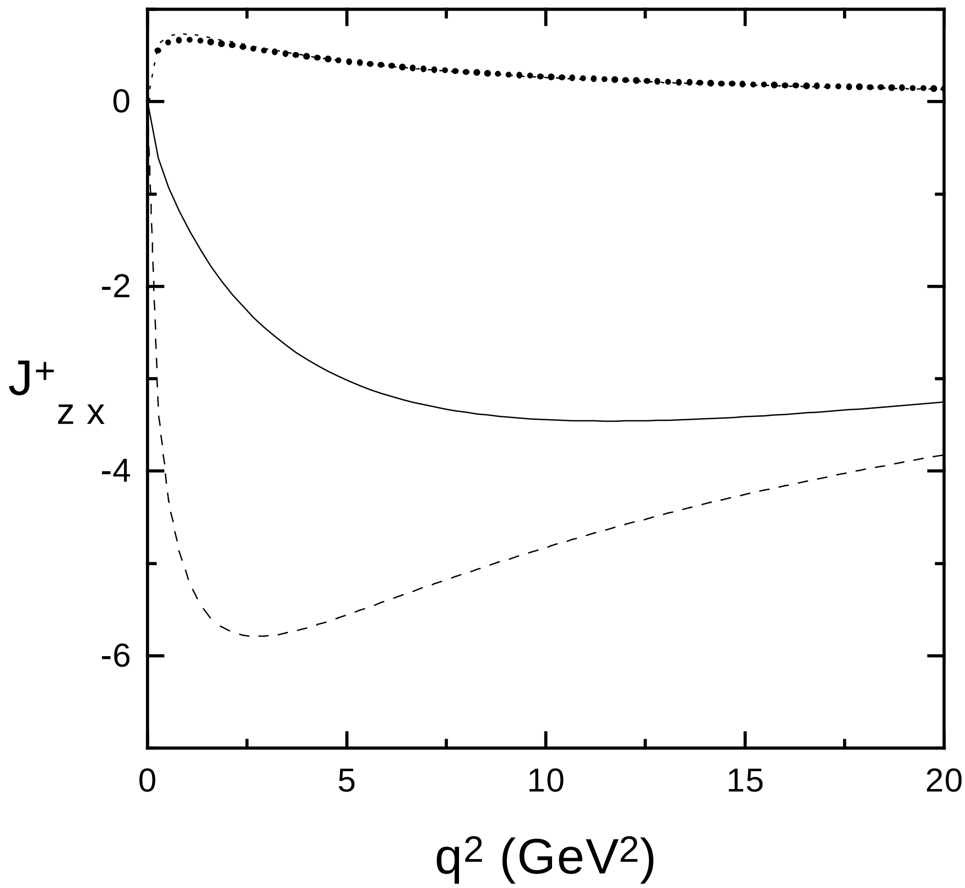

Fig. 2. Current component j^+_{zx} as a function of $|q^2|$. Curves labelled as in Fig. 1.

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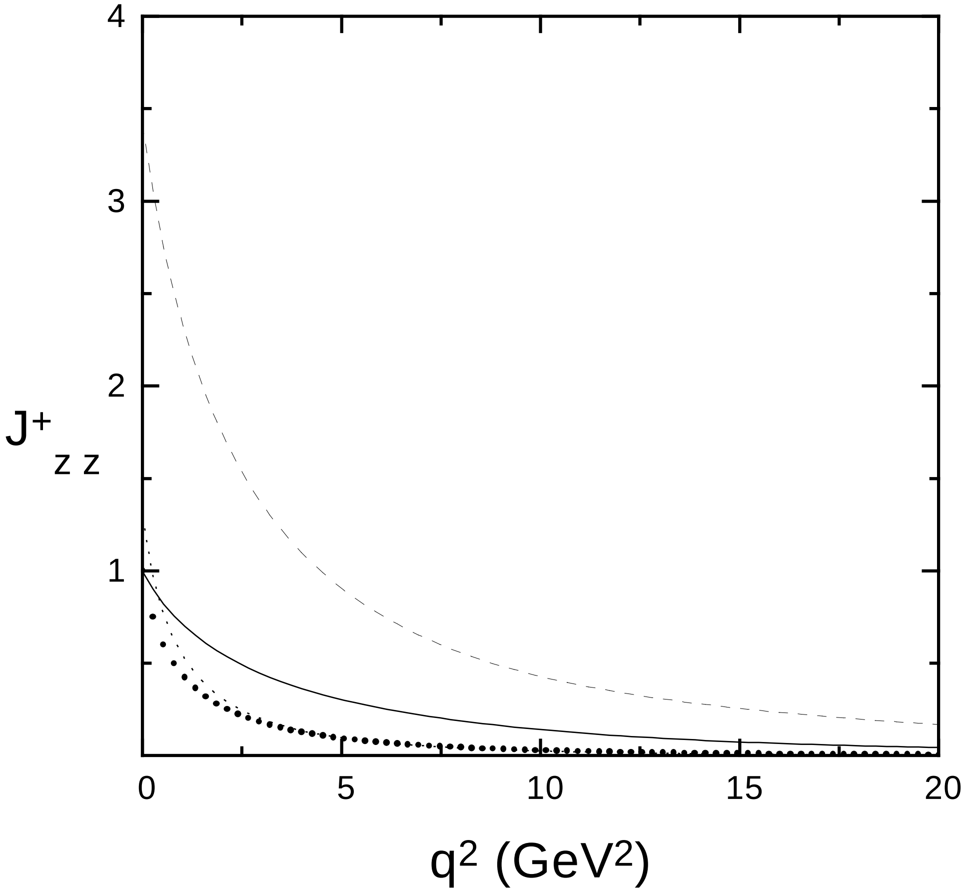

Fig. 3. Current component j^+_{zz} as a function of $|q^2|$. Curves labelled as in Fig. 1.

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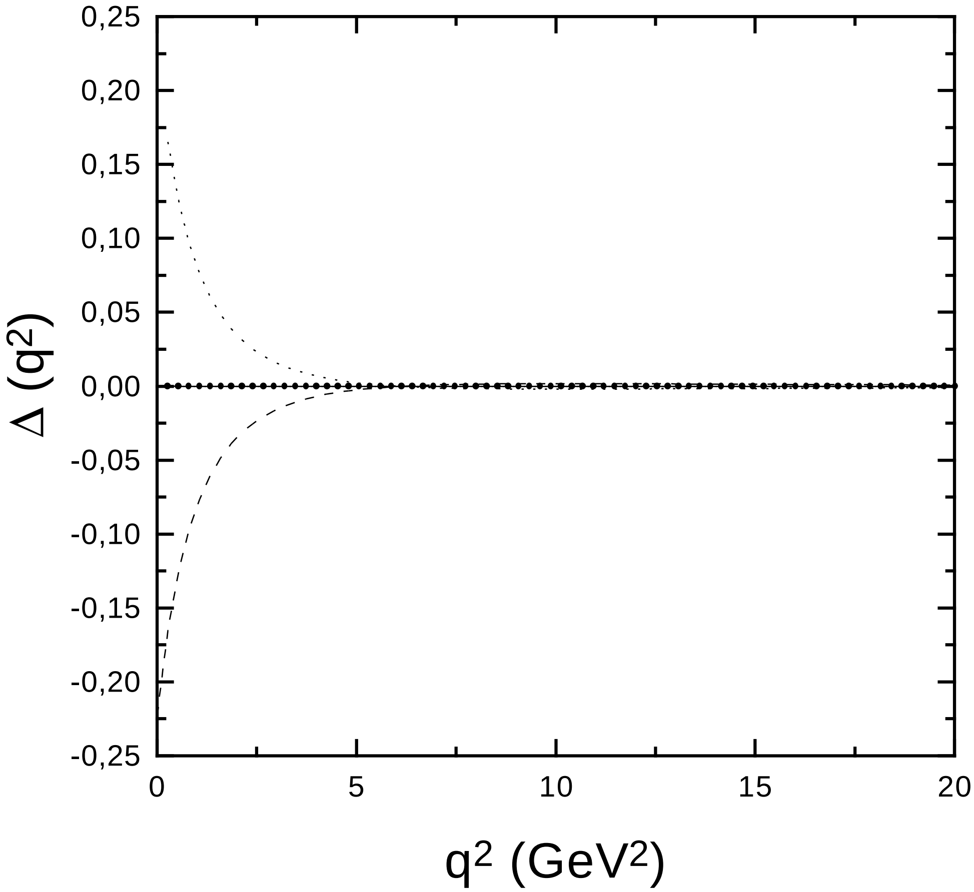

Fig. 4. Δ as a function of $|q^2|$ for γ^μ-coupling. Dashed: light-front without pair terms, dotted: pair terms, full circles: light-front including pair terms, solid: covariant calculation.

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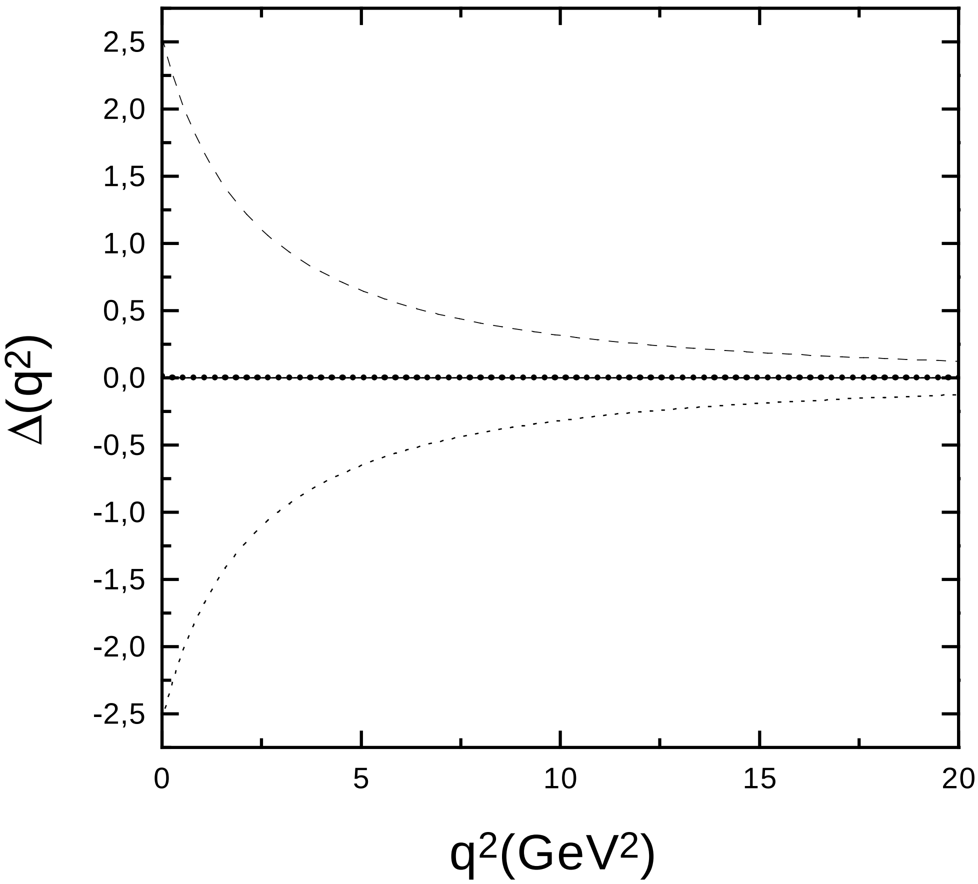

Fig. 5. Δ as a function of $|q^2|$ for derivative coupling; labelling as in Fig.4.